Day 1 – Addition and Subtraction of Whole Numbers

Goals:

I can identify 11 different addition/subtraction word problem types.

I can write grade appropriate word problems for my students.

I can describe strategies that children use to solve addition and subtraction problems.

I can modify the numbers in problems to differentiate instruction.

I can describe ways that children directly model different word problem types.

I can help students move from direct modeling to counting.

I can carry out a number talk with my students.

A Remainder of One

Five Practices for Orchestrating Productive Mathematical Discussions

Margaret S. Smith & Mary Kay Stein, NCTM & Corwin Press, 2011 www.nctm.org

1. Anticipating

- Do the problem yourself
- What are students likely to produce?
- Which problems will most likely be the most useful in addressing the mathematics?

2. Monitoring

- Listen, observe, identify key strategies
- Keep track of approaches
- Ask questions of students to get them back on track or to think more deeply

3. Selecting

- CRUCIAL STEP – what do you want to highlight?
- Purposefully select those that will advance mathematical ideas

4. Sequencing

- In what order do you want to present the student work samples?
- Do you want the most common? Present misconceptions first?
- How will students share their work? Draw on board? Put under doc cam?

5. Connecting

- Craft questions to make the mathematics visible.
- Compare and contrast 2 or 3 students' work – what are the mathematical relationships?
- What do parts of student's work represent in the original problem? The solution? Work done in the past?

Addition / Subtraction Word Problem Types

I.

Riley has 5 cookies. Seth gives him 3 more cookies.

II.

III.

IV.

Classification of Addition/Subtraction Word Problems

Although there are a number of ways that word problems can be distinguished from each other, one of the most useful ways of classifying them focuses on the types of action or relationships described in the problems. This classification corresponds to the way that children think about problems. Carpenter, et.al, 1999, p. 7

CGI Problem Type	**Various Names for Types**	**Problem Types –** students will solve problems differently depending on what is unknown and their own developmental level in mathematical understanding. (Numbers in parenthesis are for older learners.)		
Join	**Start-Change-End** problems with action	***Join (Result Unknown)*** Connie had 5 (75, 758) marbles. Juan gave her 8 (89, 895) more marbles. How many marbles does Connie have altogether?	***Join (Change Unknown)*** Connie had 5 (75, 758) marbles. How many more marbles does she need to have 13 (164, 1653) altogether?	***Join (Start Unknown)*** Connie had some marbles. Juan gave her 8 (89, 895) more marbles. Now she has 13 (164, 1653) marbles. How many marbles did Connie have to start with?
Separate		**Separate (*Result Unknown)*** Connie had 13 (164, 1653) marbles. She gave 5 (75, 758) to Juan. How many marbles does Connie have left?	***Separate (Change Unknown)*** Connie had 13 (164, 1653) marbles. She gave some to Juan. Now she has 5 (75, 758) marbles left. How many marbles did Connie give to Juan?	***Separate (Start Unknown)*** Connie had some marbles. She gave 5 (75, 758) to Juan. Now she has 8 (89, 895) left. How many marbles did Connie have to start with?
Part-Part Whole	**Part-Part Whole (Total)** ***Partner-Partner-Total*** Inactive - sets	**Part-Part Whole *(Whole Unknown)*** Connie has 5 (75, 758) red marbles and 8 (89, 895) blue marbles. How many marbles does she have?	**Part-Part-Whole *(Part Unknown)*** Connie has 13 (164, 1653) marbles. 5 (75, 758) are red and the rest are blue. How many blue marbles does Connie have?	
Compare	**Compare or Quantity-Quantity-Difference**	***Compare (Difference Quantity Unknown)*** Connie has 13 (164, 1653) marbles. Juan has 5 (75, 758) marbles. How many more marbles does Connie have than Juan?	***Compare (Compare Quantity Unknown)*** Juan has 5 (75, 758) marbles. Connie has 8 (89, 895) more marbles than Juan. How many marbles does Connie have?	***Compare (Referent Quantity Unknown)*** Connie has 13 (164, 1653) marbles. She has 5 (75, 758) more marbles than Juan. How many marbles does Juan have?

Basic problem types synthesized among various elementary curriculum materials, incorporating "just right" number choices; originally defined in Carpenter, Fennema, Franke, Levi, Empson. (1999). *Children's Mathematics: Cognitively Guided Instruction (CGI). p. 12* Portsmouth, NH: Heinemann Books, www.heinemann.com

Kindergarten Video (Mary Jo 2:52 out of 9:00)

There were 6 sharks swimming around together. 5 more sharks joined them. How many tiger sharks were there altogether?

Strategies for Addition and Subtraction

A. Join (result unknown) for $8 + 5 = ?$

a) Direct Modeling (related to counting all)

b) Counting (count on) (AS10)

c) Derived Fact (Number Sense)/ Recall (AS16)

B. Separate (result unknown) for $12 - 4 = ?$

a) Direct Modeling (AS21)

b) Counting (AS12)

c) Derived Fact (Number Sense) / Recall

C. Join (change unknown) for $7 + ? = 11$

a) Direct Modeling (AS03)

b) Counting (Grades 1 and 2 Keith and AS11)

c) Derived Fact (Number Sense) / Recall (AS14)

Practicing the Five Steps (Gr 1 and 2 Video – Part 2 (1 minute 28 seconds))

Ashley has 9 toy rockets. How many more rockets does she need to have so she'll have 17 rockets altogether?

1. **Anticipate**

2. **Monitor**
3. **Select**
4. **Sequence**

5. **Connect**

Practicing the Five Steps

Join (Result Unknown) $5 + 3 = ?$ or $8 + 7 = ?$ or $24 + 15 = ?$

1. **Anticipate**

2. **Monitor**
3. **Select**
4. **Sequence**

5. **Connect**

Number Talks

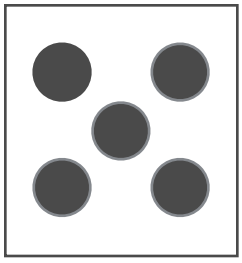
Related reading: *How Children Learn Number Concepts: A Guide to the Critical Learning Phases* by Kathy Richardson; Chapter 3

Purpose: Dot patterns and non-language context along with contextual language based problems (CGI problem solving) facilitate children's development of composite thinking. Once children have mental images then practice making this connection as it will be automatic for some but certainly not all of the students. Providing opportunities for children to solidify composite thinking is important for the development of decomposing and composing numbers.

Kindergarten

Number Talk 1

Show a regular dot pattern card



Ask, *"How many dots do you see?"*
Ask, "How do you see them?"

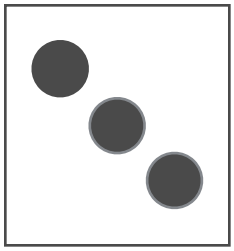
Look for responses such as 3 and 2, 4 and 1. Circle the groupings as children share.

Do similarly with 4 and 6. Later can extend to patterns for 7-9.

Number Talk 2

Teacher Materials: Domino dot pattern cards from 1-6.
Student Materials: Circle Counters

Teacher flashes a dot pattern like the one below.

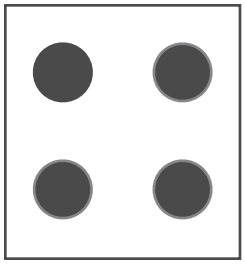


Students create the pattern using counters.

Ask, "How did you see the dots?" Children share out the ways they were able to picture the dot patterns in their head.

Number Talk 3

Flash a dot pattern from 1-5 like the one below.



Ask, “How many more to make five?”

Gather student responses.
Ask, “Where can I picture the dot on this card to make five?”
Listen for students to respond that a dot would go in the center.

Do similarly for dot pattern cards from 1-5.

Number Talk 4

Show a 5 frame like the one below.

Ask, “What do you notice?”

Listen for children to tell you the number of dots and the number of empty squares.

Do similarly for each 5 frame with 0-5 dots filled in.

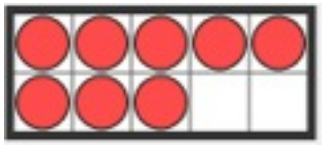
Flash a five frame similar to the one above. Ask, “What do you see?” Follow-up with the following questions if needed, “How many dots do you see?” “How many empty squares?”

Flash a five frame similar to the one above. Ask, “What do you see?” “How many more to make 5?”

Grade 1

Number Talk 1

Show a 10 frame like the one below.



Ask, “What do you notice?”
Follow-up with the following questions if needed, “How many dots do you see?” “How many empty squares?”

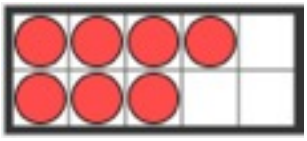
Listen for children to tell you the number of dots and the number of empty squares.

Prompt for a variety of ways to see the dots such as a group of 6 dots and 2 dots.

Do similarly for each 10 frame with 5-10 dots filled in.

Number Talk 2

Repeat using a pairs configuration such as the one below.



Ask, “What do you notice?”
Follow-up with the following questions if needed, “How many dots do you see?” “How many empty squares?”

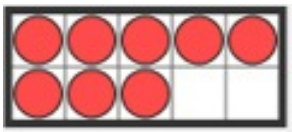
Listen for children to tell you the number of dots and the number of empty squares.

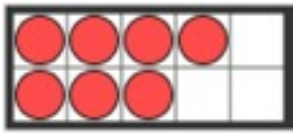
Prompt for a variety of ways to see the dots such as a group of 6 dots and 2 dots.

Do similarly for each 10 frame with 5-10 dots filled in.

Number Talk 3

Flash a ten frame similar to the ones below. Ask, “What do you see?” Follow-up with the following questions if needed, “How many dots do you see?” “How many empty squares?”





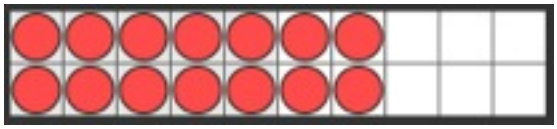
Flash a ten frame similar to the ones above. Ask, “What do you see?” “How many more to make 10?”

Grade 2

For students that have not had experience with partitioning and combining 10 and numbers up to ten, the number talks above can be done prior.

Number Talk 1

Show a 20 frame like the one below.



Ask, *"What do you notice?"*
Follow-up with the following questions if needed, "How many dots do you see?" "How many empty squares?"
If students have difficulty determining the number of dots without counting, support by asking, "How many dots are on the top row? How many dots are on the bottom row?

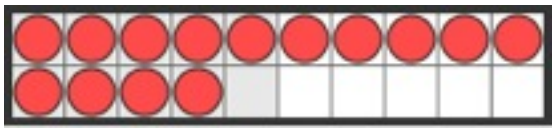
Listen for children to tell you the number of dots and the number of empty squares.

Prompt for a variety of ways to see the dots such as a group of 10 dots and 4 dots.

Do similarly for 20 frames with 10-20 dots filled in.

Number Talk 2

Show a 20 frame like the one below.



Ask, *"What do you notice?"*
Follow-up with the following questions if needed, "How many dots do you see?" "How many empty squares?"
If students have difficulty determining the number of dots without counting, support by asking, "How many dots are on the top row? How many dots are on the bottom row?

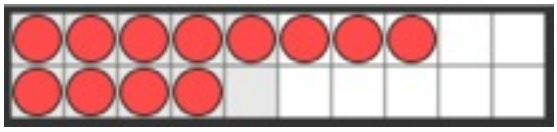
Listen for children to tell you the number of dots and the number of empty squares.

Prompt for a variety of ways to see the dots such as a group of 10 dots and 4 dots.

Do similarly for 20 frames with 10-20 dots filled in.

Number Talk 3

Show a 20 frame like the one below.

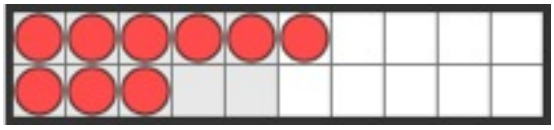


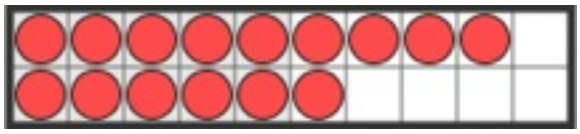
Ask, *"What do you notice?"*

Listen for children to share different ways to see and think about the number of dots.

Prompt for a variety of ways to see the dots such as a group of 8 dots and 4 dots.

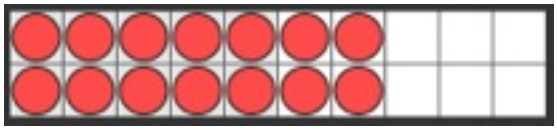
Do similarly for 20 frames such as the ones below.

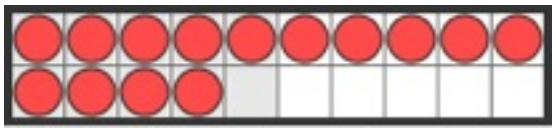




Number Talk 4

Flash a twenty frame similar to the ones below. Ask, "What do you see?" Follow-up with the following questions if needed, "How many dots do you see?" "How many empty squares?"

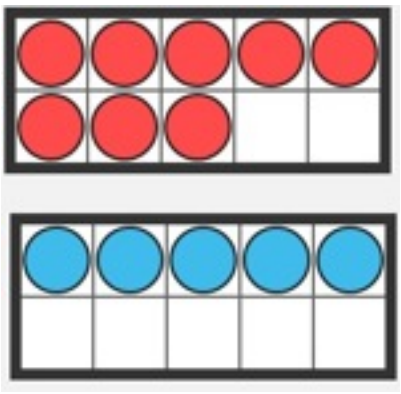




Flash a twenty frame similar to the ones above. Ask, "What do you see?" "How many more to make 20?"

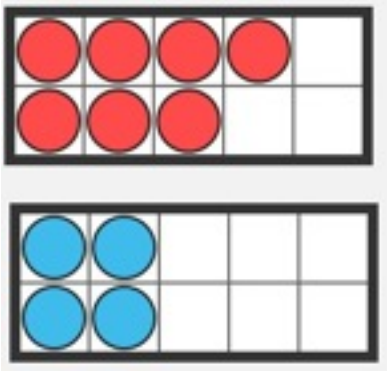
Number Talk 5

Show 2 ten frames like the ones below.



Ask, “What do you see?”
Listen for the different ways children see and think about the dots to determine a total for 8+5.

Show 2 ten frames like the ones below.



Ask, “What do you see?”
Listen for the different ways children see and think about the dots to determine a total for 7+4.