

Day 3 – Focus on Subtraction: Encouraging Children to Adopt More Sophisticated Strategies

Goals:

I can use ten-frames and rekenreks to carry out number talks with my students.

I can describe several strategies that children use to derive facts with addition and subtraction.

I can describe subtraction as how much more or less, take-away, and as a comparison.

I can record children’s thinking using a number sentence and an open number line.

I can help students move to a more sophisticated and more efficient strategy type.

Minnesota Standards

Use objects and pictures to represent situations involving combining and separating.

K.1.2.1 Use objects and draw pictures to find the sums and differences of numbers between 0 and 10.

K.1.2.2 Compose and decompose numbers up to 10 with objects and pictures.

Use a variety of models and strategies to solve addition and subtraction problems in real-world and mathematical contexts.

1.1.2.1 Use words, pictures, objects, length-based models (connecting cubes), numerals and number lines to model and solve addition and subtraction problems in part-part-total, adding to, taking away from and comparing situations.

1.1.2.2 Compose and decompose numbers up to 12 with an emphasis on making ten.

Use number sentences involving addition and subtraction basic facts to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.

1.2.2.1 Represent real-world situations involving addition and subtraction basic facts, using objects and number sentences.

1.2.2.2 Determine if equations involving addition and subtraction are true.

Demonstrate mastery of addition and subtraction basic facts; add and subtract one- and two-digit numbers in real-world and mathematical problems.

2.1.2.1 Use strategies to generate addition and subtraction facts including making tens, fact families, doubles plus or minus one, counting on, counting back, and the commutative and associative properties. Use the relationship between addition and subtraction to generate basic facts.

2.1.2.2 Demonstrate fluency with basic addition facts and related subtraction facts.

Use number sentences involving addition, subtraction and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.

2.2.2.1 Understand how to interpret number sentences involving addition, subtraction and unknowns represented by letters. Use objects and number lines and create real-world situations to represent number sentences.

2.2.2.2 Use number sentences involving addition, subtraction, and unknowns to represent given problem situations. Use number sense and properties of addition and subtraction to find values for the unknowns that make the number sentences true.

Making Sense of Subtraction

Draw pictures and write sentences to show the different ways your students might solve each problem. Rank the problems based on difficulty level (Easiest-1, Medium-2, Hardest-3).

Problem	Rank
<p style="text-align: center;">A</p> <p>Bren runs 11 miles. Jen runs 7 miles. How many more miles did Bren run than Jen?</p>	
<p style="text-align: center;">B</p> <p>Abdi has 11 cookies. He eats some of his cookies, now he has 7 cookies left. How many cookies did Abdi eat?</p>	
<p style="text-align: center;">C</p> <p>There were 11 hockey pucks in a bag. Mikko removed 7 of the pucks for practice. How many pucks were left in the bag?</p>	

11 – 7

What does the mathematical expression $11 - 7$ mean to you?

What does $11 - 7$ mean to children?

Meanings

Seeing Meanings on a Number Line



Review of Direct Modeling and Counting Strategies

The strategies should match the action or relationship described in the problem.

Problem Type	Direct Modeling	Counting
Join (Result Unknown)	Joining All	Counting On
Join (Change Unknown)	Joining To	Counting On To
Separate (Result Unknown)	Separating From	Counting Down
Separate (Change Unknown)	Separating To	Counting Down To
Compare (Difference Unknown)	Matching	No commonly used strategy corresponding to the action or relationship described in the problem.
Join (Start Unknown)	Trial and Error	Trial and Error
Separate (Start Unknown)	Trial and Error	Trial and Error
Part-Part-Whole (Part Unknown)	No commonly used strategy corresponding to the action or relationship described in the problem.	No commonly used strategy corresponding to the action or relationship described in the problem.

Adapted from page 34 from Children's Mathematics: Cognitively Guided Instruction (2nd Edition) by Carpenter et. al

Derived Facts Related to Addition and Subtraction

Recording Student Thinking using Number Sentences and Open Number Lines

$$8 + 7$$

Recording Children's Thinking Using Addition

Record strategies using number sentences and open number lines.

Number Fact	Derived Fact	
	Ten as a Benchmark	Doubles
$6 + 7$		
$8 + 9$		
$4 + 8$		
$9 + 3$		

* adapted from CGI page 46.

When you are finished, circle the derived facts that are efficient for a given problem. What patterns do you notice?

Second Grade Teacher Recording Student Thinking using Number Sentences

Number Talks Video 2.2 Addition: $16 + 15$ (3:07)

Record what the teacher writes to record student thinking.

What are the benefits of recording children's thinking?

Should children be encouraged to use notation to describe their own thinking?

Problem	Strategy child uses	How could you record thinking using a number sentence?	How could you record thinking using an open number line?
<p>AS16 (0:34) Lucy has 8 fish. She buys 5 more fish. How many fish would Lucy have then?</p>			
<p>AS14 (0:22) Keisha has 6 beads. How many more beads does she need to collect to have 13 beads altogether?</p>			
<p>AS11 (0:29) Jane has 7 trolls in her collection. For her birthday her friends give her some more trolls, now she has 11. How many trolls did her friends give her?</p>			
<p>130 (0:44) Conner 1st Grade Hannah had 25 goldfish in her aquarium. She gave her friend Sara 8 of the goldfish. How many goldfish does Hannah have now?</p>			

141 (1:08) Edgar 1 st Grade Carlos has 14 toy cars. He gives 5 toy cars to Eric. How many toy cars does Carlos have left?			
3.15 (0:20) http://smarturl.it/CM3.15 8+9			
3.24 (2:39) http://smarturl.it/CM3.24 Paco had 15 cookies. He ate 7 of them. How many cookies does Paco have left?			
3.19 (0:52) http://smarturl.it/CM3.19 4 + 8 7 + 8 8 - 2 16 - 7			

Learning Progression for Single-Digit Addition and Subtraction

Count all → Count on → Thinking strategies

Recall (1+1, 2+1, etc.) → Recall (small totals) → Recall (various totals)

12345 123
00000 000
12345 678
5+3=8

00000 000
5 678
5 678
5+3=8

Make a ten: $9 + 6 = 10 + 5 = 15$
Doubles: $6 + 7 = (6 + 6) + 1 = 12 + 1 = 13$

take away → Count up to → Thinking strategies

Recall (3 - 1 = 2, etc.) → Recall (small numbers)

1 2 3 4 5 6 7 8
~~0 0 0 0 0 0 0 0~~
1 2 3 4 5 1 2 3

a) $8 - 5 = 3$ b) 5 up to 8 is 3 more

a) Count down
7 6 5
8 4 3 left

or
7 6 5 4
8 3 3 left

Up over ten: $15 - 9$ as
 $9 + 1$ (to ten) + 5 (to 15) = 6

Down over ten: $15 - 9$ as
 5 (down to ten) + 1 (down to 9) = 6

Doubles: $13 - 7$ as
 $7 + 7 = 14$, so 6 (13 is 1 less than 14)

a) as subtractions
b) from related addition ($5 + ? = 8$ for $8 - 5$)

from Fuson

Count Up or Count Down

$$13 - 8$$

Count up by ones	Count down by ones
Count up in chunks	Count down in chunks

Word Problem Encouraging Counting Down (13 – 8)

Word Problem Encouraging Counting Up (13 – 8)

Math Talks Related to Take Away in Chunks using a Rekenrek

Model Problem: **13 take away 5**

Say: Show 13 on Rekenrek (Allow different ways).

Ask: "What is 13 take away 5? How did you do it?"

Look for different ways that children are taking away in chunks.

Partner A: **14 take away 7**

Partner B: **20 take away 8**

Partner C: **17 take away 9**

Math Talks Related to Take Away in Chunks using Number Sentences

Chantal has 12 bagels. She gives 2 bagels to Vern. How many bagels does she have left?

Model Category A:

“What is 12 take away 2?” “How did you get your answer?” “Did anyone do it a different way?”

$$12 - 2$$

$$12 - 5$$

$$15 - 5$$

$$15 - 6$$

Partner A:

$$24 - 4$$

$$24 - 5$$

$$24 - 8$$

$$24 - 9$$

Partner B:

$$35 - 10$$

$$35 - 13$$

$$35 - 20$$

$$35 - 24$$

Partner C:

$$41 - 20$$

$$41 - 23$$

$$45 - 20$$

$$45 - 28$$

Math Talks Related to Adding in Chunks using a Rekenrek

Sample Context:

Ms. Thomas has 8 pencils. She finds some pencils on the floor and now has 17 pencils. How many pencils did she find on the floor?

Say: Show 8 on Rekenrek (Allow different ways).

Ask: "How much more to get to 17?"

Look for different chunking strategies.

Partner A: **Start at 7. How much more to get 12?**

Partner B: **Start at 11. How much more to get 19?**

Partner C: **Start at 7. How much more to get 20?**

Math Talks Related to Finding the Difference in Chunks using a Rekenrek Using Single Digit Numbers

Model Problem: **What is the difference between 5 and 8?**

Ask Students: Show 5 beads on the top and 8 beads on the bottom.

Ask: Are there more beads on the top row or on the bottom row? Why? How many more beads are on the bottom row? How did you do it?

Look for different ways that the students chunk to make up the difference.

Partner A: **2 beads on top, 6 beads on the bottom.**

Partner B: **8 beads on top, 3 beads on the bottom.**

Partner C: **4 beads on top, 9 beads on the bottom.**

Math Talks Related to Finding the Difference in Chunks using a Rekenrek

Model Problem: **What is the difference between 12 and 18?**

Ask Students: Show 12 on your rekenrek (Allow different ways). Then show 18 on your (teachers) rekenrek.

Ask: Who has more beads on their rekenrek, you or me? Why? How many more beads do I have on my rekenrek than you? How did you do it?

Look for different ways that the students chunk to make up the difference.

The 6 more beads on my rekenrek is called the difference. The difference between the 12 beads on your rekenrek and the 18 beads on my rekenrek is 6?"

Partner A: Students: **8 beads on rekenrek. Teachers: 15 beads on rekenrek. "What is the difference between the 8 beads on your rekenrek and the 15 beads on my rekenrek?"**

Partner B: **Students: 13 beads on rekenrek. Teachers: 7 beads on rekenrek. "What is the difference between the 13 beads on your rekenrek and the 7 beads on my rekenrek?"**

Partner C: **Students 9 beads on rekenrek. Teachers: 20 beads on rekenrek. "What is the difference between the 9 beads on your rekenrek and the 20 beads on my rekenrek?"**

Math Talks Related to How Much More or Less using Number Sentences

Model Category A:

“What is the difference between 30 and 19?” “How did you get your answer?” “Did anyone do it a different way?”

$$30 - 19$$

$$30 - 14$$

$$30 - 24$$

$$30 - 21$$

Partner A:

$$60 - 49$$

$$60 - 29$$

$$60 - 39$$

$$60 - 19$$

Partner B:

$$44 - 39$$

$$44 - 35$$

$$44 - 29$$

$$44 - 25$$

Partner C:

$$80 - 39$$

$$80 - 68$$

$$81 - 49$$

$$81 - 58$$

Recording Children's Thinking Using Subtraction

Record strategies using number sentences and open number lines.

Number Fact	Derived Fact	
	Ten as a Benchmark	Doubles
12 – 6		
16 – 9		
11 – 4		
25 – 13		

* adapted from CGI page 46.

When you are finished, circle the derived facts that are efficient for a given problem. What patterns do you notice?

Goals for Today:

I can use ten-frames and rekenreks to carry out number talks with my students.

I can describe several strategies that children use to derive facts with addition and subtraction.

I can describe subtraction as how much more or less, take-away, and as a comparison.

I can record children's thinking using a number sentence and an open number line.

I can help students move to a more sophisticated and more efficient strategy type.

Last Page

<p>A Sam had 8 animal crackers. His sister ate some of them. Now Sam has 6 animal crackers. How many animal crackers did Sam's sister eat?</p>	<p>B Allan begins with 6 cubes. He counts out 8 additional cubes and combines the cubes. He recounts the entire set and gets 14.</p>	<p>C There are 8 birds on a branch. 6 of the birds fly away. How many birds are on the branch?</p>
<p>D Gena shows 8 cubes. She removes 2 cubes and counts to see that there are 6 left. She says that the answer is 2.</p>	<p>E Thomas has 6 goals. Zach has 8 goals. How many more goals does Zach have than Thomas?</p>	<p>F Kate guesses 5 then counts 6, 7, 8, 9, 10, 11. Says "5 is too big" and changes guess until she gets the right beginning guess of 2.</p>
<p>G Boris says 8, then says 9, 10, 11, 12, 13, 14. 14 is the answer.</p>	<p>H Danny says, "One more is 7, two more is 8. The answer is 2."</p>	<p>I Leon puts out an initial set of 15 cubes. Removes 8 cubes and counts out remaining cubes. Since the remaining amount is too big, he changes the initial amount until he gets the correct remaining amount. The answer is the initial amount.</p>
<p>J Bob has 6 flowers. Val gives him some more flowers, now he has 8 flowers. How many flowers did Val give Bob?</p>	<p>K Eli puts 8 tick marks on a piece of paper. He crosses out 6 of them and counts the remaining marks. He states that the answer is 2.</p>	<p>L Cindy has some dollars. Lou gives her 6 more dollars, now Cindy has 8 dollars. How many dollars did Cindy start with?</p>
<p>M India makes a tower of 6 cubes and another tower of 8 cubes. She puts the towers next to each other and notices that one tower is 2 cubes taller than the other. She says "2."</p>	<p>N Anne has 6 carrots. Brooke gives her 8 more carrots. How many carrots does Anne have now?</p>	<p>O Franny counts backwards saying "7, 6, 5, 4, 3, 2" showing 6 fingers. Her answer is the last number in the counting sequence, "2".</p>
<p>P There are 6 boys and some girls on the math team. There are 8 people on the team. How many girls are on the math team?</p>	<p>Q Jade puts out an initial set of 3 cubes. Adds 6 more cubes and counts 9 cubes. 9 is too big so she keeps trying different initial amounts.</p>	<p>R Maddie guesses 20 as an initial amount. Counts 19, 18, 17, 16, 15, 14, 13, 12. Says, "12 is too big." She changes her initial amount until she counts the correct remaining amount.</p>
<p>S Hannah counts one cube and says, "7 left." She counts another cube and says, "6 left." She says the answer is 2.</p>	<p>T Otis had some dog treats. He gave his dog 8 dog treats, now he has 6 left. How many dog treats did Otis start with?</p>	<p>U Cass puts out 6 cubes. He adds additional cubes, counting "7, 8" until there is a total of 8 cubes. He keeps the two additional cubes separate from the initial set. He responds 2.</p>

Answer Key for Sorting Activity

<p>Join (Result Unknown) Anne has 6 carrots. Brooke gives her 8 more carrots. How many carrots does Anne have now?</p>	<p>Joining All Allan begins with 6 cubes. He counts out 8 additional cubes and combines the cubes. He recounts the entire set and gets 14.</p>	<p>Counting On Boris says 8, then says 9, 10, 11, 12, 13, 14. 14 is the answer.</p>
<p>Join (Change Unknown) Bob has 6 flowers. Val gives him some more flowers, now he has 8 flowers. How many flowers did Val give Bob?</p>	<p>Joining To Cass puts out 6 cubes. He adds additional cubes, counting "7, 8" until there is a total of 8 cubes. He keeps the two additional cubes separate from the initial set. He responds 2.</p>	<p>Counting On To Danny says, "One more is 7, two more is 8. The answer is 2."</p>
<p>Separate (Result Unknown) There are 8 birds on a branch. 6 of the birds fly away. How many birds are on the branch?</p>	<p>Separating From Eli puts 8 tick marks on a piece of paper. He crosses out 6 of them and counts the remaining marks. He states that the answer is 2.</p>	<p>Counting Down Franny counts backwards saying "7, 6, 5, 4, 3, 2" showing 6 fingers. Her answer is the last number in the counting sequence, "2".</p>
<p>Separate (Change Unknown) Sam had 8 animal crackers. His sister ate some of them. Now Sam has 6 animal crackers. How many animal crackers did Sam's sister eat?</p>	<p>Separating To Gena shows 8 cubes. She removes 2 cubes and counts to see that there are 6 left. She says that the answer is 2.</p>	<p>Counting Down To Hannah counts one cube and says, "7 left." She counts another cube and says, "6 left." She says the answer is 2.</p>
<p>Compare (Difference Unknown) Thomas has 6 goals. Zach has 8 goals. How many more goals does Zach have than Thomas?</p>	<p>Matching India makes a tower of 6 cubes and another tower of 8 cubes. She puts the towers next to each other and notices that one tower is 2 cubes taller than the other. She says "2."</p>	<p>No commonly used strategy corresponding to the action or relationship described in the problem.</p>
<p>Join (Start Unknown) Cindy has some dollars. Lou gives her 6 more dollars, now Cindy has 8 dollars. How many dollars did Cindy start with?</p>	<p>Trial and Error Jade puts out an initial set of 3 cubes. Adds 6 more cubes and counts 9 cubes. 9 is too big so she keeps trying different initial amounts.</p>	<p>Trial and Error Kate guesses 5 then counts 6, 7, 8, 9, 10, 11. Says "5 is too big" and changes guess until she gets the right beginning guess of 2.</p>
<p>Separate (Start Unknown) Otis had some dog treats. He gave his dog 8 dog treats, now he has 6 left. How many dog treats did Otis start with?</p>	<p>Trial and Error Leon puts out an initial set of 15 cubes. Removes 8 cubes and counts out remaining cubes. Since the remaining amount is too big, he changes the initial amount until he gets the correct remaining amount. The answer is the initial amount.</p>	<p>Trial and Error Maddie guesses 20 as an initial amount. Counts 19, 18, 17, 16, 15, 14, 13, 12. Says, "12 is too big." She changes her initial amount until she counts the correct remaining amount.</p>
<p>Part-Part-Whole (Part Unknown) There are 6 boys and some girls on the math team. There are 8 people on the team. How many girls are on the math team?</p>	<p>No commonly used strategy corresponding to the action or relationship described in the problem.</p>	<p>No commonly used strategy corresponding to the action or relationship described in the problem.</p>